On preferences related to aggregative operators and their transitivity

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1 Introduction

Several definitions of preference transitivities were introduced in the past, see e.g. [2]. In this paper we deal with two new transitivity definitions, namely the additive and the multiplicative transitivity.

Definition 1. *Let $p(x,y)$ be a preference function. It fulfills the additive transitivity if*

$$\left(p(x,y)-\frac{1}{2}\right)+\left(p(y,z)-\frac{1}{2}\right)=p(x,z)-\frac{1}{2},\qquad \forall x,y,z\in[0,1].$$

Definition 2. *Let $p(x,y)$ be a preference function. It fulfills the multiplicative transitivity if*

$$\frac{p(y,x)}{p(x,y)}\frac{p(z,y)}{p(y,z)}=\frac{p(z,x)}{p(x,z)},\qquad \forall x,y,z\in[0,1].$$

These definition correspond respectively to the nilpotent and the strict classes of fuzzy operators. We give two new preference functions which fulfill these transitivities. Both of them originate from the concept of aggregative operators [1]. The pseudo-associative additive aggregative operator is the mean operator

$$m(x,y)=f_m^{-1}\left(\frac{f_m(x)+f_m(y)}{2}\right),$$

where $f_m:[0,1]\to[0,1]$ is a nilpotent generator function. The associative multiplicative aggregative operator is

$$a(x,y)=f_a^{-1}\left(f_a(x)\,f_a(y)\right),$$

where $f_a:[0,1]\to[0,\infty]$ is a strict generator function. We define a preference function to be

$$p(x,y)=o(n(x),y),$$

where $o(x,y)$ is either an additive or a multiplicative aggregative operator and $n(x)$ is its corresponding strong negation. We prove that the (additive and multiplicative) preferences are strongly related to the Łukasiewicz and the Dombi operators, respectively. We show that the generator functions of the preference functions can only be the generators of these operators.

Theorem 1. *The preference function*

$$p(x,y) = f^{-1}\left(\frac{1}{2}\left(f(y) - f(x)\right) + f(\nu)\right)$$

has additive transitivity if and only if its generator function is $f(x) = cx$ *and so*

$$p(x,y) = \frac{y - x + 1}{2}.$$

Theorem 2. *The preference function*

$$p(x,y) = f^{-1}\left(f(\nu)\frac{f(y)}{f(x)}\right)$$

has multiplicative transitivity if and only if its generator function is $f(x) = \frac{1-x}{x}$ *and so*

$$p(x,y) = \frac{1}{1 + \frac{1-y}{y}\frac{x}{1-x}}.$$

We also show some interesting properties of these preferences and their corresponding aggregative and negation operators.

Proposition 1. *The following identities hold:*

- $p(x,y) = n(p(y,x))$
- $p(x,y) = p(n(y), n(x))$
- $p(x,y) = n(p(n(y), n(x)))$
- $p(o(x_1,\ldots,x_n), o(y_1,\ldots,y_n)) = o(p(x_1,y_1),\ldots,p(x_n,y_n))$

References

1. József Dombi. Basic concepts for a theory of evaluation: the aggregative operator. *European Journal of Operations Research*, 10:282–293, 1982.
2. T. Tanino. Fuzzy preference orderings in group decision making. *Fuzzy Sets and Systems*, 12(2):117–131, 1984.