Összehasonlíthatósági mértékekről

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Szegedi Tudományegyetem

Döntés

	c_1	c_2	$\cdots$	c_m
a	x_1	x_2	$\ldots$	x_m
b	y_1	y_2	$\ldots$	y_m
	w_1	w_2	$\ldots$	w_m

$$X_j = u_j(x_j)$$

$$Y_j = u_j(y_j)$$

- Utility

$$aggr(X_1, w_1; \ldots; X_m, w_m) = A$$

$$aggr(Y_1, w_1; \ldots; Y_m, w_m) = B$$

- Outranking

$$p_1 = p(x_1, y_1)$$

$$\vdots$$

$$p_m = p(x_m, y_m)$$

$$P(a,b) = aggr(p_1, w_1; \ldots; p_m, w_m)$$

ELECTRE I

- $p_i \in \{0,1\}$
- Concordance
  - Ha y_i jobb, mint x_i, akkor p_i=1 : $x_i \le y_i \Rightarrow p_i = 1$
  - $p(x,y) = 1 - p(y,x) \quad x \neq y$

    $p(x,x) = 1$

    $P(a,b) = \sum w_i p_i$
  - P(a,b) és P(b,a) között nincs összefüggés

Módosított ELECTRE I

- $$p(x,y) = \begin{cases} 1 & \text{ha} \quad x < y \\ 1/2 & \text{ha} \quad x = y \\ 0 & \text{ha} \quad x > y \end{cases}$$

$$P(a,b) = 1 - P(b,a)$$

Discordance

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	c_1	c_2	$\cdots$	c_m
a	x_1	x_2	$\dots$	x_m
b	y_1	y_2	$\dots$	y_m
	u_1	u_2	$\dots$	u_m

$$d(x_i, y_i) = \begin{cases} 0 & \left|x_i - y_i\right| \geq u_i \\ 1 & \left|x_i - y_i\right| < u_i \end{cases}$$

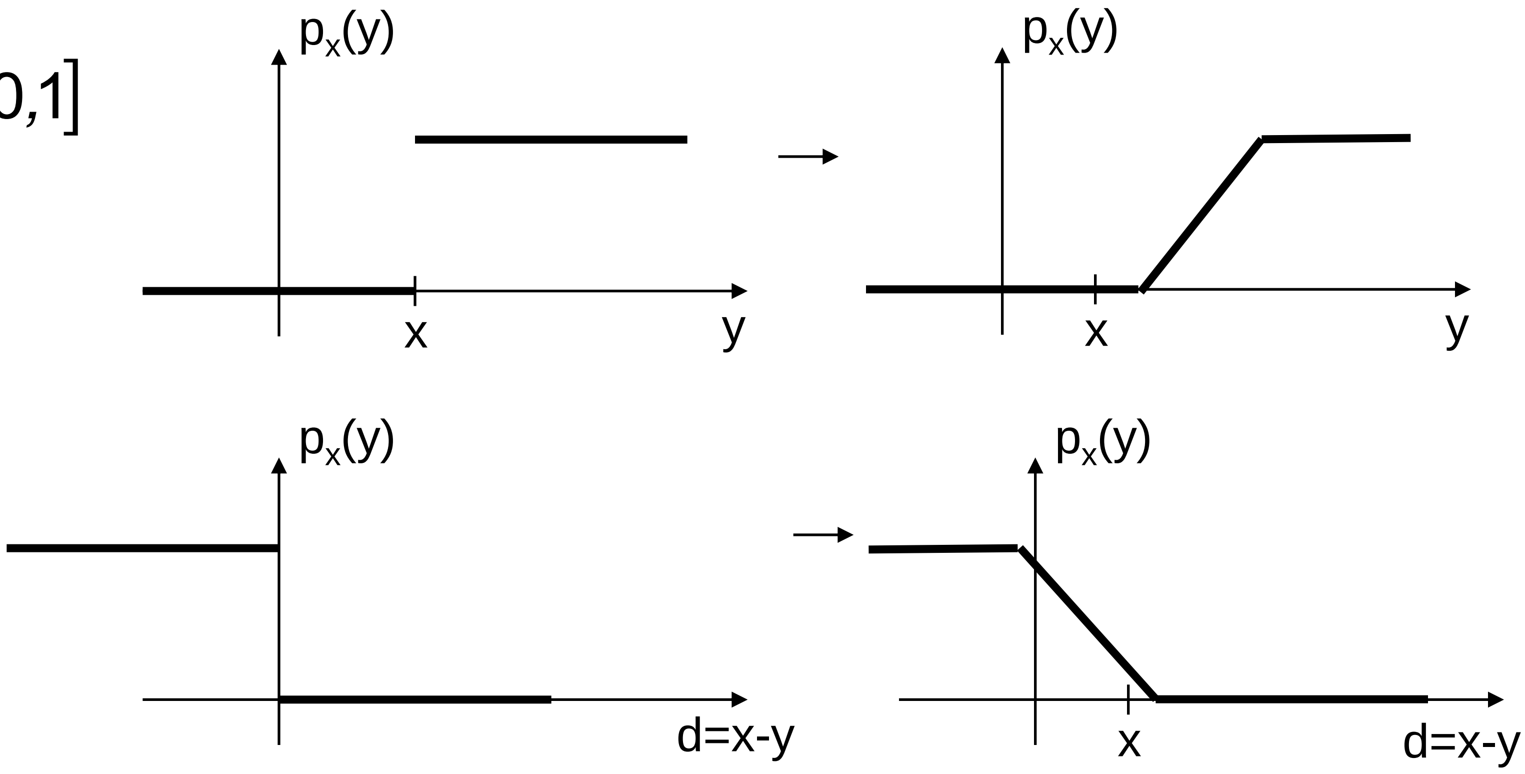
$$D(a,b) = \prod d(x_i, y_i)$$

- Outranking

$$S(a,b) = F(P(a,b), D(a,b))$$

ELECTRE II, III, IV…

- $p_i \in [0,1]$



- $F(P,\underline{d}) = P \cdot \prod_{d_j > P} \frac{1-d_j}{1-P}$

Általános koncepció

- $p^*(x,y) = \frac{y-x+1}{2} \quad (\text{truth}\{x<y\})$

- $p^*(x,y) = 1/2 \qquad x = y$

  $p^*(x,y) = 1 \qquad x = 0 \wedge y = 1$

  $p^*(x,y) = 0 \qquad x = 1 \vee y = 0$

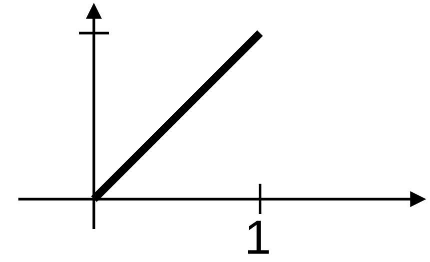
  $p^*(x,y) = 1 - p(y,x)$

  $p^*(x,y) = p(1-y, 1-x)$

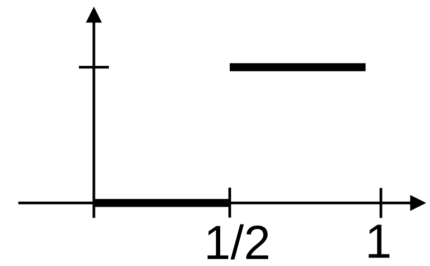
- $p(x,y) = \tau(p^*(x,y)) \qquad \tau : [0,1] \to [0,1]$

Speciális esetek

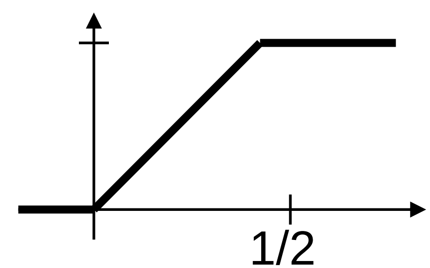
- $\tau(x) = x \quad \sum w_i x_i$ utility



- $\tau(x) = \begin{cases} 0 & x < 1/2 \\ 1 & x \geq 1/2 \end{cases}$ ELECTRE I

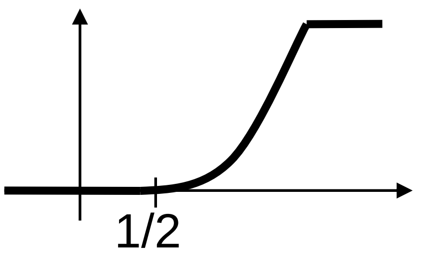


- $\tau(x) = \begin{cases} 1 & q < x \\ \dfrac{x-p}{q-p} & p < x < q \\ 0 & x < p \end{cases}$ ELECTRE II

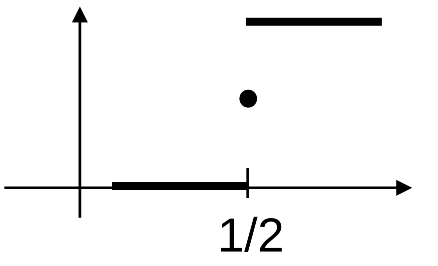


Speciális esetek (folyt.)

- $$\tau(x) = \begin{cases} 1 & q < x \\ f_p(x) & p < x < q \\ 0 & x < p \end{cases}$$ PROMETHEE



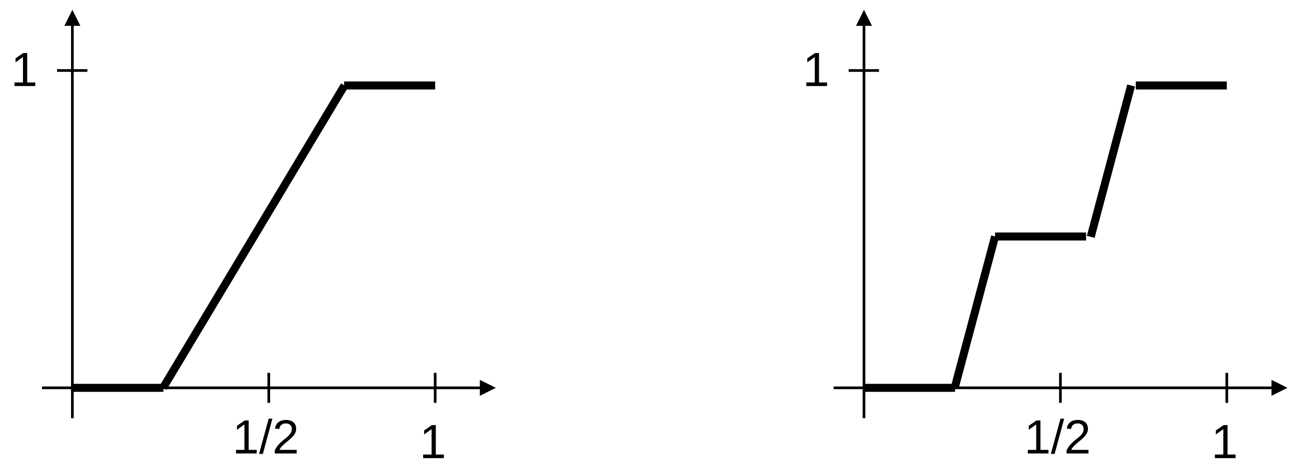
- $$\tau(x) = \begin{cases} 0 & x < 1/2 \\ 1/2 & x = 1/2 \\ 1 & x > 1/2 \end{cases}$$ lexicographic



$$w_i = \frac{1}{2^i}$$

Szimmetria

- $\tau(x) = \tau(1 - x)$
- $P(a,b) = 1 - P(b,a)$

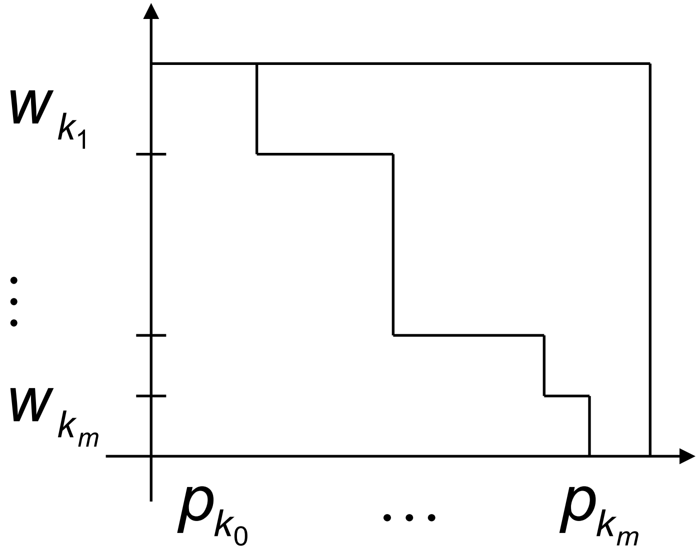


G függvény

- $k_1, \ldots, k_m$ legyen 1,…,m permutációja, hogy

$$0 = p_{k_0} < p_{k_1} \le \ldots \le p_{k_{m+1}} = 1$$

$$\sum w_{k_i} = 1$$



$$G_{a,b}(x) = \sum_{i:x<p_{k_i}} w_{k_i}$$

$$P(a,b) = \int G_{a,b}(x)dx$$

Discordance

- ELECTRE I
  - $d(x,y) = \begin{cases} 0 & \text{ha } x > y \\ \dfrac{y_i - x_i}{\delta} & \end{cases}$ $\qquad \delta = \max(y_i - x_i)$
  - $d(x,y) = \begin{cases} 0 & \text{ha } x \geq y \\ \dfrac{y_i - x_i}{\delta} & \end{cases}$ Vincke
  - $d(x,y) = \begin{cases} 0 & \text{ha } \quad x \geq y \\ 1 & \text{ha } y - x > d \end{cases}$ Roy

Discordancia (folyt.)

- ELECTRE III $d(x,y) = \begin{cases} 0 & \text{ha} \quad y\text{-}x < p \\ \dfrac{y-x-p}{v-p} & \text{ha} \quad p \le y\text{-}x \le v \\ 1 & \text{ha} \quad y\text{-}x > v \end{cases}$

- PROMETHEE (Brans)

$$d(x,y) = \begin{cases} 1 \quad \text{ha} \quad \sum p(a_i,x) > \sum p(a_j,x) \quad \text{és} \quad \sum p(x,a_i) > \sum p(x,a_j) \\ 1 \quad \text{ha} \quad \sum p(a_i,x) < \sum p(a_j,x) \quad \text{és} \quad \sum p(x,a_i) < \sum p(x,a_j) \\ 0 \qquad \text{különben} \end{cases}$$

- SCDAS (Lewandowski, Wiesbiczki)

$$p_{j_1} > p_{j_2} > \ldots > p_{j_n} \qquad \Delta p_{j_k} = p_{j_k} - p_{j_{k-1}}$$

$$c(k) = 16(k-1)^2(n-1-k)^2/(n-2)^4$$

$$d(a_i,a_j) = \sum_{k=2}^{m-2} c(k)\Delta p_{j_k}$$

Fuzzy

- operátorok
  - asszociativitás
  - idempotencia

$$o(x,y) = f^{-1}(f(x)+f(y))$$

$$\bar{o}(x,y) = f^{-1}\left(\frac{1}{2}(f(x)+f(y))\right)$$

- Halmazhoztartozási fgv. karakterisztikus fgv.

$$\mu(x) \qquad \chi(x)$$

- Fuzzyság mértéke: $d(\mu(x),\chi(x))$
- Például

$$\mu(x_1),\mu(x_2),\ldots,\mu(x_n)$$

$$F(\mu) = -c\sum_{i=1}^{n}\mu(x_i)\log\mu(x_i) + (1-\mu(x_i))\log(1-\mu(x_i))$$

Fuzzyság mértéke

- Tulajdonságok
  - permutációtól való függetlenség
  - $\mu(x) \in \{0,1\} \Rightarrow F(\mu) = 0$
  - $\mu(x) = 1/2 \Rightarrow F(\mu) = 1$
  - $F(\mu) = F(1-\mu)$
  - Ha μ_1 élesebb, mint μ_2 akkor $F(\mu_1) \geq F(\mu_2)$

$$\left|\mu_1 - \frac{1}{2}\right| \geq \left|\mu_2 - \frac{1}{2}\right| \quad \text{azaz} \quad \begin{array}{lll} \mu_1 \geq \mu_2 & \text{ha} & \mu_2 \geq 1/2 \quad \text{és} \\ \mu_1 \leq \mu_2 & \text{ha} & \mu_2 \leq 1/2 \end{array}$$

Fuzzyság mértéke (folyt.)

- Direkt szorzat $(\mu_1 \times \mu_2)(x, y) = \mu_1(x) \cdot \mu_2(y)$
- Fuzzy halmaz ereje $P(\mu) = \sum \mu(x_i)$
- Tulajdonságok

$$F(\mu_1 \times \mu_2) = H(P(\mu_1), P(\mu_2), F(\mu_1), F(\mu_2))$$
$$F(\mu_1 \times \mu_2) = F(\mu_2 \times \mu_1)$$

Fuzzyság mértéke (folyt.)

- Entrópia
- $4\sum \mu(x_i)(1-\mu(x_i))$
- $\frac{1}{n}\sum K(\mu(x_i))$
  - K(0) = 0   K(1) = 0   K(1/2)=1
- 2min(x,1-x)

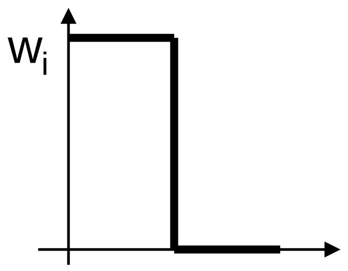
Fuzzyság mértéke és az operátorok

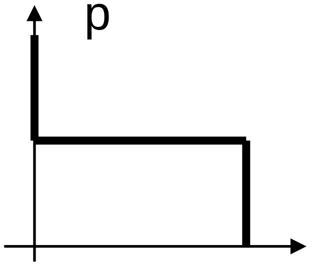
- $K(x) = \dfrac{1}{\bar{c}(1/2, 1/2)} \bar{c}(x, 1-x)$
- $\bar{c}(x, y) = \dfrac{1}{1 + \frac{1}{2}\left(\left(\frac{1-x}{x}\right) + \left(\frac{1-y}{y}\right)\right)} \Rightarrow x(1-x)$
- Entrópia $\rightarrow$ operátor (?)

  $c(x, y) = max(0, x + y - 1)$ Lukasiewicz $\rightarrow$ limeszben
- Tétel (Ebanks): F(μ) = 4μ(1-μ)
- Tétel (Sander): F(μ) = μ log μ + (1- μ)log(1- μ)

Fuzzyság mérték és összehasonlíthatóság mérték

- G(x):





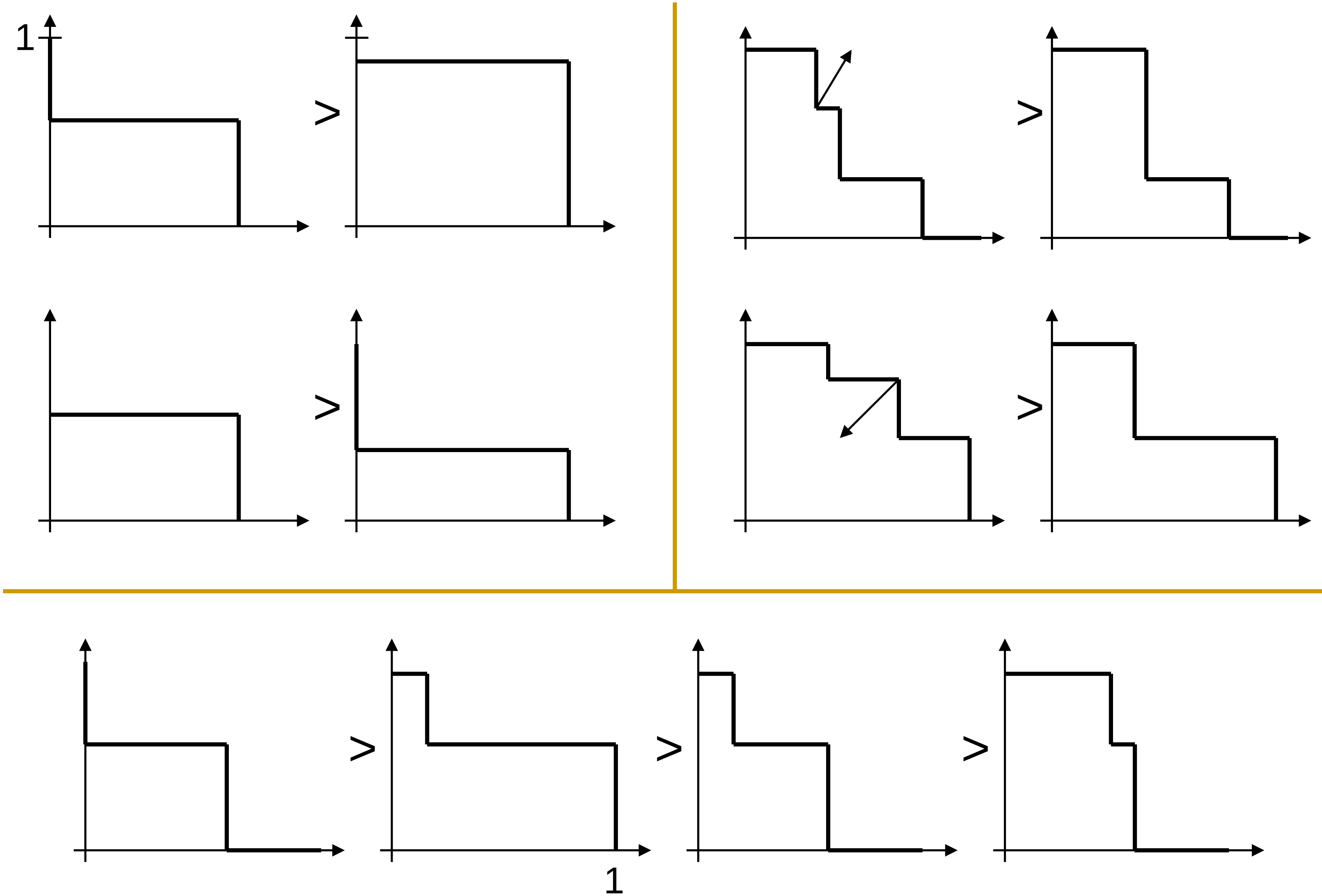
w_i = szavazók
p = preferencia (felosztási arány)

incomparability = 0

50 % pro
50 % con

incomparability = 1

  - független wi-k permutációjától
  - Ha G1 élesebb, mint G2 akkor G1 összehasonlíthatóbb, mint G2
  - G és 1-G ugyanolyan mértékig összehasonlítható $F(G) = F(1-G)$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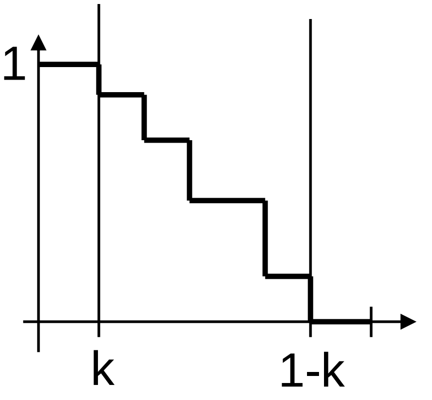
Incomparability mérték

- $d(a_i, a_j) = d(G_{ij}(x))$
- $d(G_{ij}(x)) = \int_0^1 F(G_{ij}(x))\, dx$
- $$\begin{aligned} d(G) = \; & F(w_{i_1})(p_{i_2} - p_{i_1}) + F(w_{i_1} + w_{i_2})(p_{i_3} - p_{i_2}) + \\ & \vdots \\ & + F(w_{i_1} + \ldots + w_{i_{n-1}})(p_{i_n} - p_{i_{n-1}}) \end{aligned}$$
- $F(x) = 4x(1 - x)$

ELECTRE

- k < p(a,b) < 1-k



- Preference $\int G(x)dx$ terület (area)
- Comparability $\int F(G(x))dx$ alak (shape)

- Az összehasonlíthatóság függ w_i-től és a preferencia struktúrájától
- Alkalmazás: klaszterezés!

Köszönöm a figyelmet!

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