Cognitive Maps Based on Pliant Logic

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Abstract

In this paper we present a tool for description, and for simulation of dynamic system. Our starting point is the aggregation concept, which were developed for multicriteria decision making. Using a continuous logic operator and proper transformation of sigmoid function we build positive and negative effects. From the input with the aggregation operator we calculate the output effect. This algorithm is comparable with the concept of fuzzy cognitive maps. We show this new technique, which could be much more efficient then the FCM.

1 Introduction

Handling sophisticated systems we face serious difficulties, because we have to approach dynamic systems. Modeling a dynamic system can be hard in a computational sense. In addition formulating a system with mathematical model may be difficult, even impossible.

Developing the model requires effort and specialized knowledge. Usually the system involves complicated causal chains, which may be nonlinear. It should be mentioned to, that numerical data may be hard to get and even uncertain. Our approach overcome the above mentioned difficulties. It is qualitative approach, where enough to know rough description of the system and not necessary deep expert knowledge.

First type of this approach proposed by Kosko (Kosko 1988), and called Fuzzy Cognitive Map (FCM). FCM's are hybrid methods lie in some sense between fuzzy systems and neural networks. Knowledge is represented in symbolic manner, states, processes and events. All type of this information has numerical values.

FCM allows to perform qualitative simulations and experiment width the model. Compared FCM either expert system and neural networks it has good properties as it is relative easy to use for representing structured knowledge and the inference can be computed by numeric matrix operation instead of applying rules.

2 Basic Concept of Pliant Cognitive Map (PCM)

In this paper we make closer the FCM concept to the real world modeling. We will use the cognitive maps as a formal way of representing knowledge and modeling decision making, which was introduced by Axelrod (Axelrod 1976). Kosko used fuzzy values and matrix multiplication to calculate the next stage of the concepts representing by cognitive map. Instead of values we will use time dependent functions like an impulse function representing the positive and negative influences. An other improvement is dropping the matrix multiplication concept, because on one hand it not works using continuous logic (or fuzzy logic), where the truth value has 1 and the false is 0 as usually used and the negative effect build by negation. The other hand more general operators could be more effective. Logic and the cognitive map model correspond each other, and much more easy for the expert to build up and construct the system and from the identified system extract the knowledge.

Combining cognitive maps with logic can avoid many of knowledge extraction problems instead of rule based system. The classical knowledge representation in expert system is made through a decision tree. This form of knowledge presentation can not model the dynamic behavior of the world.

The cognitive map describe the whole system by a graph showing the cause effects along concepts. It is an directed graph with feedback, that the world as a collection of concepts and causal influences between the concepts. From logic point of view the causal concepts are unary operators of a continuous valued logic, which may negation operators in the case of inhibition effects. The value of node reflects the degree of the activity of the system at a particular time. Concept values are expressed on a normal range denoting a degree of activation rather than an exact quantities value. The inverse of the normalization could express the values coming from the real world. In spite of Fuzzy Cognitive Maps we do not use thresholds to forced the values between 0 and 1. The mapping is a variation of the "fuzzification" process in fuzzy logic, but this destroys getting the possibility of quantitative results. In pliant logic we use only continuously strict monotonously increasing functions, and the inverse function always corresponds the real world values with the logical values.

In FCM the causal relationship are expressed by either positive or negative signs ordered by different weights. As we mentioned this will be replaced by unary operators.

Let $\{C_1, \ldots, C_m\}$ be concepts. Let's define over the concepts a directed graph. A directed edge W_{ij} from concept C_i to concept C_j measures how much C_i causes C_j. $W_{ij} \in [0,1]$ where $1/2$ is the neutral value, 0 is maximum negative and 1 is maximal positive influence or causality (In FCM $W_{ij} \in \{-1, 0, 1\}$):

- $W_{ij} > 1/2$ indicates direct (positive) causality between concepts C_i and C_j. That is the increase (decrease) in the value of C_i leads to increase (decrease) on the value of C_j.
- $W_{ij} < 1/2$ indicates inverse (negative) causality between concepts C_i and C_j. That is the increase (decrease) in the value of C_i leads to decrease (increase) on the value of C_j.
- $W_{ij} = 1/2$ indicates no relationship between C_i and C_j.

In pliant case W_{ij} depends on time (t) i.e. $W(t) = (W_{ij}(t))_{n\times n}$ The activation level a_i of concept C_i calculated by an iteration process. In FCM is $a_j^n = f\left(\sum_{i=1}^n w_{ij} a_j^0\right)$ where a_i^n is the new activation level of concept C_i at time $t+1$, a_i^0 is the activation level of concept C_i at time t and f is threshold function. FCM has the advantage that we get the new state vector a by multiplying the previous state vector a by the edge matrix W showing the effect of the change in the activation level of one concept to another concept. In the pliant concept we aggregate the influences instead of summing up the values. The result is always remain between 0 and 1, so we can avoid to normalization as an artificial step. The aggregation is pliant logic is general operation, which contain the conjunctive operators and disjunctive operators as well. Depends on the parameter - called neutral value - of aggregation operator we can build logical operators (Dombi operators). Using PCM (Pliant Cognitive Maps) can be used to answer "what if" question based on an initial scenario. Let a_0 the initial state vector. Repeatedly calculate with the aggregation operator the new state until the system convergence (i.e. $|a_i^0 - a_i^n| < \epsilon$). We get the resulting equilibrium vector, which provides the answer to the "what-if" question. The PCM can be used all the areas covered by FCM.

3 Aggregation and its Properties

Beside the developed logical operators in fuzzy theory appears non logical operators. The reason was insufficiency of using either conjunctive or disjunction operators for real world situation [Zimmermann].

Genral class of the fuzzy operators is called t-norm and t-conorm (disjunctive case). Denoting by $c(x, y)$ the conjunctive operator and $d(x, y)$ the disjunctive operator then

$$c(x, y) \leq \min(x, y)$$

$$d(x, y) \geq \max(x, y)$$

In the real world situation often occur that an aggregation value $a(x, y)$ is

$$\min(x, y) \leq a(x, y) \leq \max(x, y)$$

The rational form of an aggregation operator is [Dombi]:

$$a(x_1, \ldots, x_n) = \frac{1}{1 + \left(\frac{1-\nu_0}{\nu_0}\right)\left(\frac{\nu}{1-\nu}\right)^n \prod_{i=1}^n \left(\frac{1-x_i}{x_i}\right)}$$

or

$$a(x_1, \ldots, x_n) = \frac{1}{1 + \left(\frac{\nu_*}{1-\nu_*}\right)^{n-1} \prod_{i=1}^n \left(\frac{1-x_i}{x_i}\right)}$$

Where ν is the neutral value.
The corresponding negation function is

$$n_\nu(x) = \frac{1}{1 + \dfrac{1-\nu_0}{\nu_0} \cdot \dfrac{1-\nu}{\nu} \cdot \dfrac{x}{1-x}}$$

$$n_{\nu_*}(x) = \frac{1}{1 + \left(\dfrac{1-\nu_*}{\nu_*}\right)^2 \cdot \dfrac{x}{1-x}}$$

The aggregation operator is axiomatically based and it has several good properties as

1. defined on $(0, 1)$ and the values are also in $(0, 1)$
2. associativity
3. continuously
4. strictly monotonously increasing
5. continuous on $[0, 1)$ interval
6. $a(0, 0) = 0$ and $a(1, 1) = 1$

Aggregation is connected to negation operators. The proportion of that negation are

1. defined on $(0, 1)$ and the values are also in $(0, 1)$
2. $n(0) = 1$
3. $n(1) = 0$
4. continuous
5. strictly decreasing function
6. involutiv (the double negation is the identity) $n(n(x) = x$

We can ordered to a negation a ν_* fixed point such $n(\nu_*) = \nu_*$ or defined ν_0 the so called common threshold and neutral value than $n(\nu) = \nu_0$. Usually $\nu_0 = 1/2$.

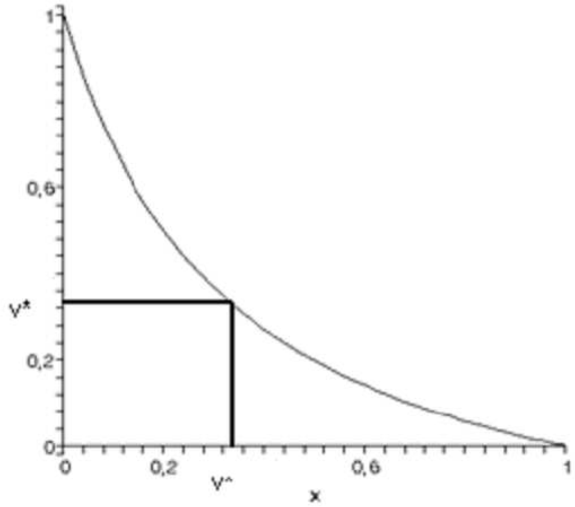

Figure 1: Negative function with ν_* threshold

The negative function and aggregation operator is closely related. It can be seen easily that

1. $n(a(x, y)) = a(n(x), n(y))$
2. $a(x, n(x) = \nu_0$
3. $a(x, \nu_0) = x$

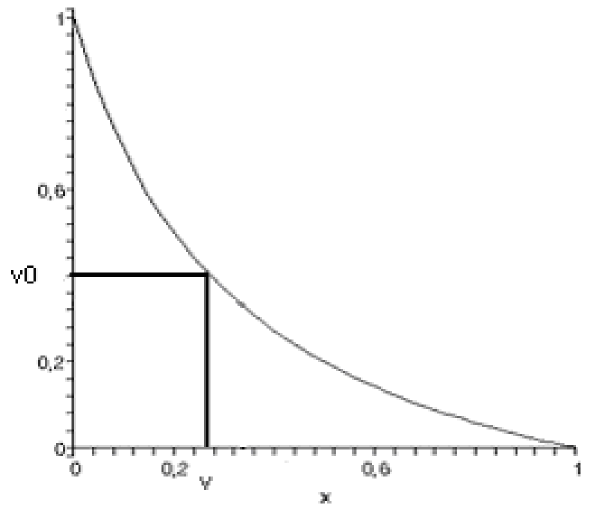

Figure 2: Negative function width ν neutral value and ν_0 threshold

The properties of the aggregation are natural: 1, aggregating positive values and negating it is the same if we aggregating negative values, 2, aggregating positive and negative values we get the neutral values back, 3, aggregating x with the neutral value we get back x.

The property ensures, that we can replace the sum function with the aggregation function of the pliant logic. The neutral value here is ν_0 instead of 0 using in FCM. The neutral value corresponds the aggregation width the logical connectives to. It can be proved that

1. if $x, y \leq \nu$ then $a(x, y) \leq \min(x, y)$
2. if $x, y \geq \nu$ then $a(x, y) \geq \max(x, y)$
3. if $x \leq \nu \leq y$ then $\min(x, y) = x \leq a(x, y) \leq y = max(x, y)$

First means that if the values are less then neutral value then the aggregation is conjunction. Second means that if the values are larger then neutral value then the aggregation is disjunction.Third means that the aggregation of positive and negative values are always between the two values.

We can model conjunctive and disjunctive operator with the aggregation operator. If is close to 0 then operation has disjunctive character and if is close to 1 then the operation has conjunctive character. From this property it can be seen, that using aggregation we have much more possibilities instead of using the sum function in FCM. Changing in the nodes the neutral value different operation can be carried out.

4 Producing Influences

Our starting point is the sigmoid function.

$$\sigma_a^{(\lambda)}(t) = \frac{1}{1 + e^{-\lambda(t-a)}}$$

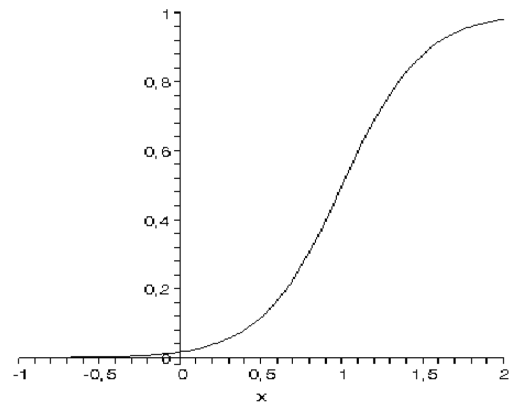

Figure 3: Sigmoid function

It is easy to see:

1. $\sigma_a^{(\lambda)}(a) = 1/2$
2. $\sigma'_a(a) = \lambda$
3. $\sigma_a^{(-\lambda)}(t) = 1 - \sigma_a^{(\lambda)}(t)$

The sigmoid function natural way maps the values to the $(0, 1)$ interval. Positive (negative) influences can be build with $\sigma_a^{(\lambda_1)}(t)$, $\sigma_b^{(\lambda_2)}(t)$ and conjunctive operator where $\lambda_1 > 0$, $\lambda_2 < 0$ and $a < b$ ($\lambda_1 > 0$, $\lambda_2 <$ and $b < a$).

Using the Dombi operator (Dombi) and sigmoid

$$c(x_1, w_1, \ldots, x_n, w_n) = \frac{1}{1 + \left(\sum_{i=1}^{n} w_i \left(\frac{1-x_i}{x_i}\right)^{\alpha}\right)^{1/\alpha}}$$

function with proper weights we can get

$$\sigma_{a,b}^{\lambda_1,\lambda_2}(t) = \frac{1}{1 + \frac{\lambda_2}{\lambda_1+\lambda_2} \cdot e^{-\lambda_2(t-a)} + \frac{\lambda_1}{\lambda_1+\lambda_2} \cdot e^{-\lambda_1(t-b)}}$$

where $\lambda_i > 0$ and $a < b$. For the aggregation we have to transform $P(t)$ the positive influence into $[1/2, 1]$ interval and $N(t)$ the negative influence into $[0, 1/2]$ interval:

- $P(t) = \frac{1}{2}(1 + \sigma_{a,b}^{\lambda_1,\lambda_2}(t))$
- $N(t) = \frac{1}{2}(1 - \sigma_{a,b}^{\lambda_1,\lambda_2}(t))$

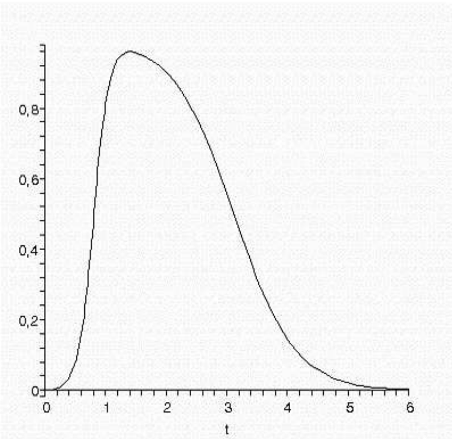

Figure 4: Asymmetrical positive influence on $[0, 1]$

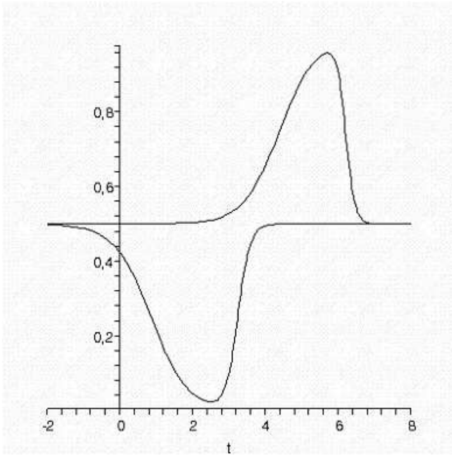

Figure 5: Positive and negative influences proposed to aggregation

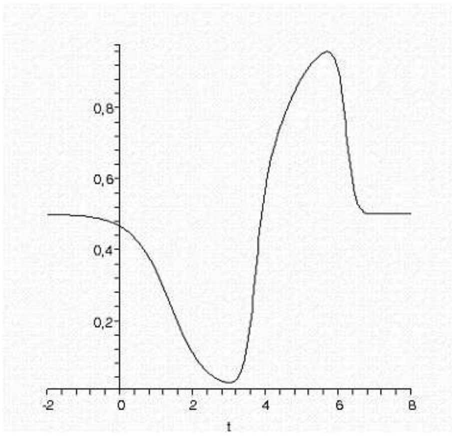

Figure 6: Aggregation of positive and negative influences

5 Value Transformation

To complete the Pliant Cognitive Map we have to give unary transformation on the values produced by the aggregation. In the pliant logic the general form of the unary operators are:

$$\pi_{v_{ij}}^{\lambda_{ij}}(x) = \frac{1}{1 + \left(\frac{1-\nu_{ij}}{\nu_{ij}} \cdot \frac{1-x}{x}\right)^{\lambda_{ij}}}$$

The sign of λ_{ij} means that it is a positive or negative influence, the value of λ mean the sharpness and ν_{ij} the expectation level between C_i and C_j.

6 Construction PCM

It is easy to build PCM. The following steps sould be carries out:

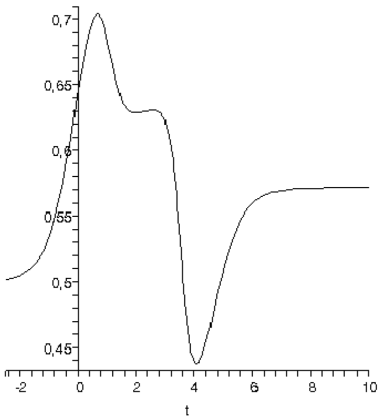

Figure 7: Aggregation of four influences

1. Collect the concepts
2. Define the expectation values of the nodes (i.e. threshold values of the aggregations)
3. Build a cognitive map (i.e. draw a directed graph between the concepts)
4. Define the influences (i.e. are they positive or negative) and determine the expectation values
5. Use the proper function to the input nodes (i.e. application of $P(t)$ and $N(t)$)

The system now ready to make simulation test. We are developing a program to test the system. First we are studying artificial situation, and this shows that the system is very flexible, and easy to adopt to various situation. For the real world application we invent learning process finding the best parameter. This lead to a nonlinear problems.

7 Conclusion

We propose a new type of numerical calculus modeling complex systems based on positive and negative influences. This concept is similar to FCM, but the functions are quite different. It is based on a continuous valued logic and all the parameters has semantic meaning. We are working a real world application and on an effective learning of the parameters of the system.

References

Author Biographies

JÓZSEF DOMBI was born in Zalaegerszeg, Hungary, and went to the University of Szeged, where he studied mathematics. His scientific degree: M. Sc. degree mathematician (1972), Ph.D. degree (1977, Summa cum laude on Hydrogen transfer reaction), CSC degree Candidate of the mathematical science (1994, The structure of the operators of fuzzy sets in the respect of multiple criteria decisions).

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