

# A General Class of Fuzzy Operators, the DeMorgan Class of Fuzzy Operators and Fuzziness Measures Induced by Fuzzy Operators

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## Abstract

In this paper we examine operators which can be derived from the general solution of functional equations on associativity. We define the characteristics of those functions  $f(x)$  which are necessary for the production of operators. We shall show, that with the help of the negation operator for every such function  $f(x)$  a function  $g(x)$  can be given, from which a disjunctive operator can be derived, and for the three operators the DeMorgan identity is fulfilled. For the fulfillment of the DeMorgan identity the necessary and sufficient conditions are given.

We shall also show that an  $f_\lambda(x)$  can be constructed for every  $f(x)$ , so that for the derived  $k_\lambda(x, y)$  and  $d_\lambda(x, y)$   $\lim_{\lambda \rightarrow \infty} k_\lambda(x, y) = \min(x, y)$  and  $\lim_{\lambda \rightarrow \infty} d_\lambda(x, y) = \max(x, y)$ .

As Yager's operator is not reducible, for every  $\lambda$  there exist an  $\alpha$ , for which, in case  $x < \alpha$  and  $y < \alpha$ ,  $k_\lambda(x, y) = 0$ .

We shall give an  $f(x)$  which has the characteristics of Yager's operator, and which is strictly monotone.

Finally we shall show, that with the help of all those  $f(x)$ , which are necessary when constructing a  $k(x, y)$ , an  $F(x)$  can be constructed which has the properties of the measures of fuzziness introduced by A. De Luca and S. Termini. Some classical fuzziness measures are obtained as special cases of our system.

*Keywords:* fuzzy operator, DeMorgan identity, fuzziness measure, axiom system